Mathematical Language Routines

Developing Students’ Voices and Sense Making

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Illustrative Mathematics

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Santa Barbara Unified School District
Let’s get to know each other!

Who teaches or works with teachers...

- PK - 2?
- 3 - 5?
- 6 - 8?
- 9 - 12?
- Post secondary?

Why are you here?
Learning Goals

➢ Understand that learning mathematics is a language-demanding activity for all students
➢ Understand how MLRs support mathematics sense-making and language development
➢ Engage in and prepare MLRs
Let’s Do Math

A company claims that their new bottle holds 25% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold?
Language demands?

Prompts:

● How did you use language to engage in this task?
● How did you make your thinking visible?

A company claims that their new bottle holds 25% more laundry soap. If their original container held 53 fluid ounces of soap, how much does the new container hold?
Mathematical Language Demands

*It matters for all students*

- Reading
- Writing
- Representing
- Conversing
- Listening
- Speaking

(Adapted from Aguirre & Bunch, 2012)
1. Stronger and Clearer Each Time
2. Collect and Display
3. Clarify, Critique, Correct
4. Information Gap
5. Co-Craft Questions
6. Three Reads
7. Compare and Connect
8. Discussion Supports

MLRs are structured but adaptable formats for amplifying, assessing, and developing students' language.
What are instructional routines?

“Instructional routines are specific and repeatable designs for learning that support both the teacher and students in the classroom.”
Let’s Do a Mathematical Language Routine
A company claims that their new bottle holds 25% more laundry soap. If their original container held 60 fluid ounces of soap, how much does the new container hold?
Clarify, Critique, Correct

Partners Clarify (1 minute)
- “I notice ...” “What do you notice?”
- “I wonder...” “What do you wonder?”

Partners Critique (2 minutes)
- “This reasoning is strong because ...”
- “What might make this reasoning stronger is ...”

Individuals Correct (2 minutes)
- “I can build onto this response by ...”
- “What else could I do/show?”
“We noticed ...“

“We wondered...”

- Using percentages
  - Double number line
  - Didn't show work
  - Filled out Double Number Line with a 25%
  - Bottom line goes 4/15
  - Top line adding 25% each time

- How did they get 15 on the bottom?
- Why is 25% is before the 100% if it's 25% more?
Critique

“This reasoning is strong because …”

“What might make this reasoning stronger is …”

- I understand it ... it's going up by 25's and 15's.
- The Double Number Line is easier because you can use both numbers.
- We need how much is 25%.
- $\sqrt{60}$ will help them find.
- The original bottle of 60 ounces.
- This is to help get 25%.
- The bottom number line need the starting 0 ounces.

We can finish this double number line because we need to extend to the both lines.
60 fluid ounces

+ 25% more

0% 25% 50% 75% 100%
The new container contains 75 fluid ounces. 100% + 25% = 125%. 60 + 15 = 75 fluid ounces. 25% is 15 more. 100% is 60. 25% is 15. 125% is 75.
Unpack the Routine

How does “Clarify, Critique, Correct”

- support language demands of the task?
- support development of mathematical understanding?
Unpack the Routine

In this routine:

- What is the teacher doing?
- What are students doing?
Mathematical Language Routine: Clarify, Critique, Correct

Purpose: To give students a piece of mathematical writing that is not their own to analyze, reflect on, and develop. The intent is to prompt student reflection with an incorrect or incomplete written argument, and for students to improve upon the written work by clarifying thinking, offering critique and then correcting the initial response.
Clarify, Critique, Correct

1. **PRESENT** a partial or broken argument, explanation, or solution method.

2. **PROMPT** partners to:
   - **Clarify**: the reasoning in the initial response
   - **Critique**: Analyze the response in light of partner’s own understanding of the problem
   - **Correct**: Partners work collaboratively / individually to improve the initial response

3. **SHARE**: Partners share out their drafts of an improved response and refine as needed during whole group discussion.
A Partnership

**Design Principles**

*To Promote Mathematical Language Use and Development in Curriculum and Instruction*

1. Support sense-making
2. Optimize output
3. Cultivate conversation
4. Maximize linguistic and cognitive meta-awareness
Connecting to the UL/SCALE Guiding Principles

Principle 3 CULTIVATE CONVERSATION: Strengthen the opportunities and supports for constructive mathematical conversations (pairs, groups, and whole class).

Conversations are back-and-forth interactions with multiple turns that build up ideas about math. Conversations act as scaffolds for students developing mathematical language because they provide opportunities to simultaneously make meaning and communicate that meaning (Mercer & Howe, 2012; Zwiers, 2011). They also allow students to hear how other students express their understandings. When students have a reason or purpose to talk and listen to each other, interactive communication is more authentic. For example, when there is an “information gap,” in which students need or want to share their thoughts (which are not the same), students have a reason or purpose in talking and listening to each other.

During effective discussions, students pose and answer questions, clarify what is being asked and what is happening in a problem, build common understandings, and share experiences relevant to the topic. As mentioned in Principle 2, learners must be supported in their use of language, including within conversations, to make claims, justify claims with evidence, make conjectures, communicate reasoning, critique the reasoning of others, and engage in other mathematical practices – and above all, to make mistakes. Meaningful conversations depend on the teacher using lessons and activities as opportunities to build a classroom culture that motivates and values efforts to communicate.
Students are Engaged in Mathematical Language Development When...

- Extending discourse
- Discussing complex problems
- Giving explanations
- Constructing arguments
- Making conjectures
- Reading complex sentences
- Stating assumptions
- Using vocabulary in context
Routines in the Mathematics Lessons Support Students’... 

Mathematical Sense Making 

Mathematical Language Development 

Simultaneously
A Double Challenge for English Learners

“(The English learner must) learn how to effectively employ a new language in an academic setting, while learning through that language the knowledge and skills in multiple disciplines.”

(UL/SCALE 2014)
Let’s Do Another Mathematical Language Routine
What do you notice?
What do you wonder?
Pick 2 equations. Describe how the two equations are related

A

\[3 + 6 + 6 + 6 = 4x + 6\]

\[21 + x = 6 + 5x\]

\[x = 3.75\]
What do you notice? What do you wonder?

Mathematical Routine: Stronger and Clearer Each Time

1. Pre-write
2. Think time
3. Pair share
4. Repeat with new partners
5. Revise pre-write

<table>
<thead>
<tr>
<th>Your Name</th>
<th>Prompt:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pick two equations. Escoge dos ecuaciones.</td>
</tr>
<tr>
<td></td>
<td>Describe how the two equations are related. Describe cómo se relacionan las dos ecuaciones.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Initial Thinking</th>
<th>Prompt:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Include diagrams, number sentences, words, etc.</td>
<td>Pick two equations. Escoge dos ecuaciones.</td>
</tr>
<tr>
<td>Describe how the two equations are related. Describe cómo se relacionan las dos ecuaciones.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Share #1</th>
<th>Prompt:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Write down 1 or 2 words before you switch partners</td>
<td>¿Did it? ¡Lo hice!</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Share #2</th>
<th>Prompt:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Say &quot;because&quot; to justify your steps</td>
<td>¿Did it? ¡Lo hice!</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>New Thinking</th>
<th>Prompt:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remember: stronger &amp; clearer!</td>
<td>¿Did it? ¡Lo hice!</td>
</tr>
<tr>
<td>Más fuerte y más claro!</td>
<td></td>
</tr>
</tbody>
</table>

Handout 5
Pick 2 equations. Describe how the two equations are related.

3 + 6 + 6 + 6 = 4x + 6

21 + x = 6 + 5x

x = 3.75
<table>
<thead>
<tr>
<th>Initial Thinking</th>
<th>Se relacionan sumando 3 veces 6 para sumarlo el 3+6+6+6=4\times6 por que cuentan los números en la parte de la izquierda (triangles on the left side)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pensamiento inicial</td>
<td>la ecuación es la misma que la otra.</td>
</tr>
<tr>
<td>Share #1</td>
<td>Share #2</td>
</tr>
<tr>
<td>Write down 1 or 2 words before you switch partners</td>
<td>Say “because” to justify your steps</td>
</tr>
<tr>
<td>Compartir #1</td>
<td>Compartir #2</td>
</tr>
<tr>
<td>Escriba 1 o 2 palabras antes de cambiar de pareja</td>
<td>Di “porque” para justificar tus pasos</td>
</tr>
<tr>
<td>Did it! ¡Lo hice!</td>
<td>Did it! ¡Lo hice!</td>
</tr>
</tbody>
</table>
Revised Response

How are the equations related?
Why does the match make sense?

Because las dos ecuaciones son los mismo cada vez que lo somemos, they are related because un can use both of them to solve a problem. They are equivalent because the two Z pentagons have the same.
Unpack the Routine

How does “Stronger and Clearer Each Time”

- support language demands of the task?
- support development of mathematical understanding?
Unpack the Routine

In this routine:

- What is the teacher doing?
- What are students doing?
Purpose: To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output (Zwiers, 2014). Pairs borrow and use the language, ideas, and justifications each time. Responses become:

- Stronger (often longer) with better justifications and examples,
- Clearer with more precise terms and linked, organized, complete sentences.
After experiencing both routines:

- How is learning mathematics a language demanding activity for all students?
- How do these routines support mathematical sense making and language development simultaneously?
- How do these routines empower students?
Let’s Prepare a Mathematical Language Routine
Implementing Mathematical Language Routines

Which of these routines will I try with my students?

- 3 Reads
- Clarify, Critique, Correct
- Stronger and Clearer Each Time

I want to try ___ because...
As you prepare to develop your own students’ voices and sense making, what are some things you might consider as you prepare to facilitate the routine with your students?
Consider Students’ (and your) Cognitive Load

Routine is new

Routine
Mathematics
UL/SCALE: Mathematical Language Routines

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MLRs are structured but adaptable formats for amplifying, assessing, and developing students' language.
Available for fall implementation

Version 3.1415 available for fall implementation including Spanish translations and EL supports for teachers

Coming July 2021

Visit illustrativemathematics.org to learn more.
THANK YOU
For Exploring Mathematical Language Routines

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