Illustrative Mathematics

Coaching for Content:

The Top 3 Elementary Math Topics That Must Be Taught Differently

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"Knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably."

~ Deborah Ball

Three Guesses

What 3 math topics must be taught differently in K-5?

? ? ?

Topic 1:

Addition Fluency within 20

Topic 1: Addition Fluency Within 20

- Why this topic?
- What are the expectations for fluency?
- What have we been doing?
- What can we do we get kids there?

Principles to Action

Build procedural fluency from conceptual understanding

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems

A Careful Progression to Fluency

	Within 5	Within 10	Within 20	Within 100	Within 1,000	Within 1,000,000
Grade K						
Grade 1						
Grade 2						
Grade 3						
Grade 4						

What Do The Standards Say?

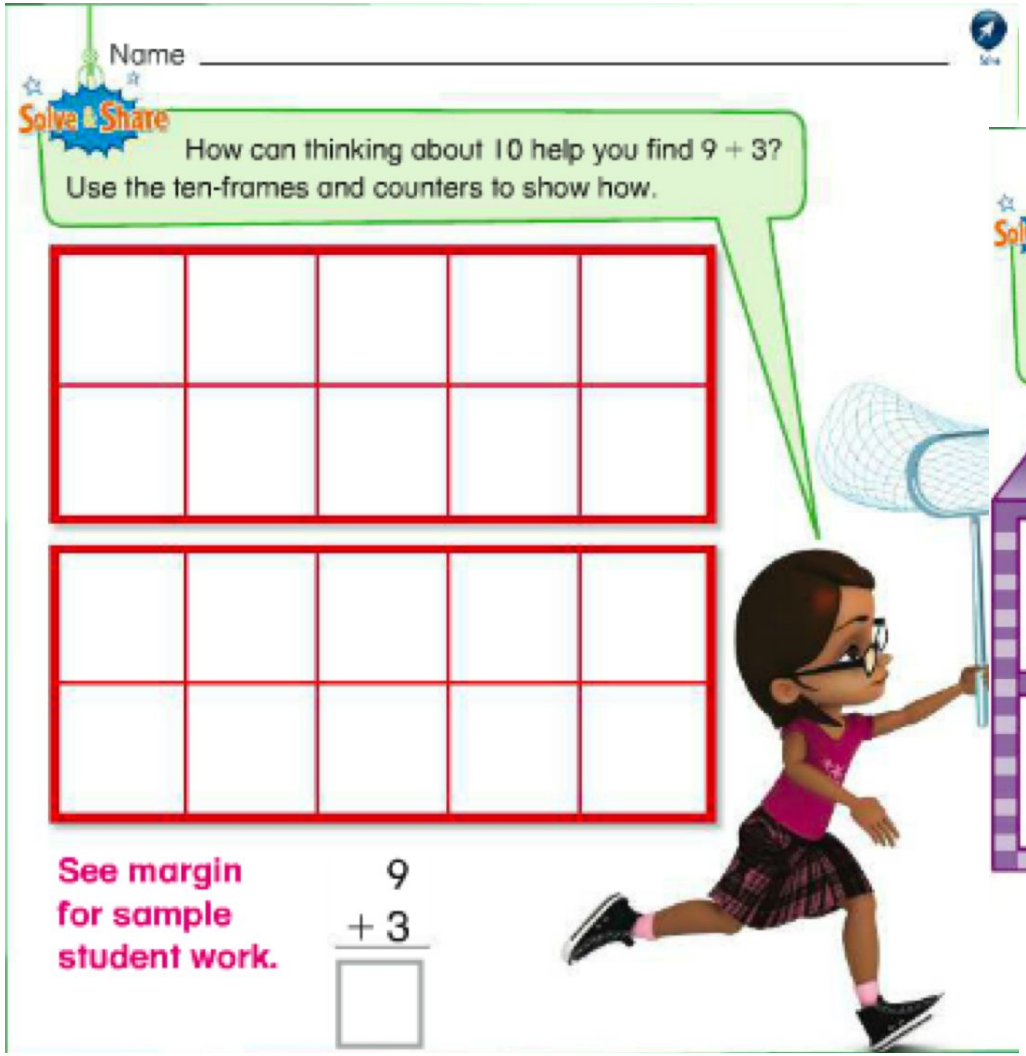
1.OA.C Add and subtract within 20.

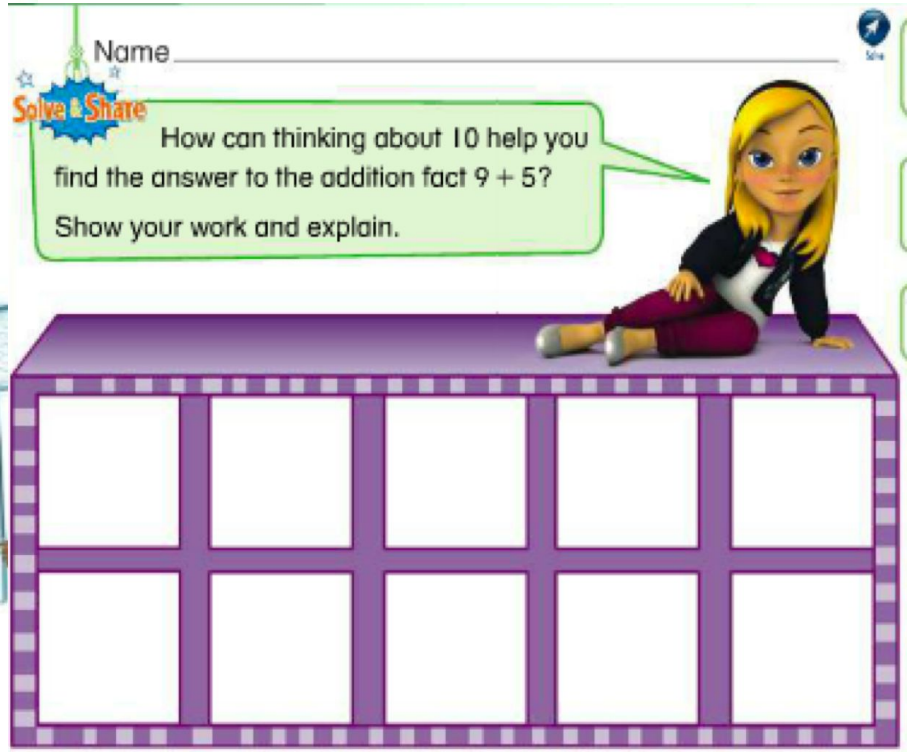
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1= 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

2.OA.B Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Grade 1 or 2?


Name
Solve & Share
How can thinking about 10 help you find 9 + 3?
Use the ten-frames and counters to show how.
See margin for sample student work.
9
+ 3


Name
Solve & Share
How can thinking about 10 help you find the answer to the addition fact 9 + 5?
Show your work and explain.

Addition Facts Within 20

0+0	1+0	2+0	3+0	4+0	5+0	6+0	7+0	8+0	9+0	10+0	11+0	12+0	13+0	14+0	15+0	16+0	17+0	18+0	19+0	20+0
0+1	1+1	2+1	3+1	4+1	5+1	6+1	7+1	8+1	9+1	10+1	11+1	12+1	13+1	14+1	15+1	16+1	17+1	18+1	19+1	
0+2	1+2	2+2	3+2	4+2	5+2	6+2	7+2	8+2	9+2	10+2	11+2	12+2	13+2	14+2	15+2	16+2	17+2	18+2		
0+3	1+3	2+3	3+3	4+3	5+3	6+3	7+3	8+3	9+3	10+3	11+3	12+3	13+3	14+3	15+3	16+3	17+3			
0+4	1+4	2+4	3+4	4+4	5+4	6+4	7+4	8+4	9+4	10+4	11+4	12+4	13+4	14+4	15+4	16+4				
0+5	1+5	2+5	3+5	4+5	5+5	6+5	7+5	8+5	9+5	10+5	11+5	12+5	13+5	14+5	15+5					
0+6	1+6	2+6	3+6	4+6	5+6	6+6	7+6	8+6	9+6	10+6	11+6	12+6	13+6	14+6						
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0+16	1+16	2+16	3+16	4+16																
0+17	1+17	2+17	3+17																	
0+18	1+18	2+18																		
0+19	1+19																			
0+20																				

6+1	Counting on
6+0	Property of 0
2+3	Addition within 10
4+6	Partners of 10
8+5	Making 10
10+7	Meaning of teen numbers
13+4	Place value computation
10+10	Link between count sequence and place value

Topic 1: Addition Fluency within 20

Addition Worksheets

1, 3, or 5 Minute Drills for Addition Worksheets

An addition drill is a worksheet with all of the single digit problems for one operation on one page. A student who has memorized all of the single digit addition problems should be able to work out the 100 problems correctly in 5 minutes, 60 problems in 3 minutes, or 20 problems in 1 minute.

Choose the numbers for the top addend

☑ 0 ☑ 1 ☑ 2 ☑ 3 ☑ 4 ☑ 5 ☑ 6
☑ 7 ☑ 8 ☑ 9 ☑ 10 ☑ 11 ☑ 12 ☑ 13
☑ 14 ☑ 15 ☑ 16 ☑ 17 ☑ 18 ☑ 19 ☑ 20
All Clear

Choose the numbers for the bottom addend

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6
☑ 7 ☐ 8 ☐ 9 ☐ 10 ☐ 11 ☐ 12 ☐ 13
☐ 14 ☐ 15 ☐ 16 ☐ 17 ☐ 18 ☐ 19 ☐ 20
All Clear

Type of Addition Drill

○ 20 Problems for a 1 Minute Drill
◉ 60 Problems for a 3 Minutes Drill
○ 100 Problems for a 5 Minute Drill

If you know the sum, just write it down. If not, then find the sum by making ten.

5 + 9	7 + 4	6 + 8	8 + 4	7 + 6	9 + 4
7 + 7	3 + 8	9 + 6	8 + 3	4 + 7	6 + 9
7 + 5	8 + 7	2 + 9	5 + 7	9 + 9	6 + 7

Zimba, “How I see Addition Facts” *Colorado Mathematics Teacher,* Fall 2016

Grade 1: Addition within 20

Lesson	Page	Learning Content	Workbook (Exercise Number)
1	31-33	• Introduction • Addition calculation of two one-digit numbers using the making-ten strategy (9 + 4)	21
2	34	• Calculation method for using the addend decomposition when an augend is 9	22
3	35	• Calculation method for using the addend decomposition when an augend is 9	23
4	36	• Calculation method for using the addend decomposition when an augend is 8	24
5	37	• Calculation method for using the addend decomposition when an augend is 7	25
6	38	• Calculation practices using addend decomposition when augends are 9, 8, or 7	26
7	39	• Addition calculation of two one-digit numbers using the making-ten strategy (4 + 9)	27
8	40	• Calculation method for using the augend decomposition when an addend is 9	28
9	41	• Calculation practices using augend decomposition when the addends are 9, 8, or 7	29
10	42	• Calculation practices that involve decision between addend decomposition or augend decomposition	30
11	43	• Calculation practices that involve decision between addend decomposition or augend decomposition	31
12	44	• Application problems for addition calculation (2 "add to" problems)	32
		ut together" problems)	33
		ut together" problem & 1	34
		dd to" problem & 1 "put	35
		dd to" problem & 1 "put	36
			37
			38
			39
		ons	40

au·gend

/ˈô ˌjend/

noun MATHEMATICS · COMPUTING

the number to which an addend is added.
"we can begin with the combination in which both the addend and the augend are 0's but the carry bit from the previous column is a 1"

A Focused Approach

Draft of IM Elementary Curriculum

Grade 1 Unit 3

- Section 1: 10 as a Unit
- Section 2: Composing 10 to Add and Subtract
- Section 3: Using Facts I Know to Add and Subtract
- Section 4: Applying Our Strategies to Word Problems

Topic 2:

Fractions as an Extension of work with Whole Numbers

Topic 2: Fractions as an Extension of Work with Whole Numbers

- Why are fractions on the list?
- How does the language of college- and career-ready standards shift how we teach fractions?
- Examples

Why Fractions?

"The coherence and sequential nature of mathematics dictate the foundational skills that are necessary for the learning of algebra. The most important foundational skill not presently developed appears to be proficiency with fractions. The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected."

Source: Final Report of the National Mathematics Advisory Panel (2008, p. 18)

What has changed?

FRACTIONS

WHOLE NUMBERS

"Children must adopt new rules for fractions that often conflict with well-established ideas about whole number"

Bezuk & Cramer, 1989

What has changed?

WHOLE NUMBERS → FRACTIONS

3.NF.A Develop understanding of fractions as numbers.

4.NF.B Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

5.NF.B Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

What do you notice?
What do you wonder?

Grade 3 Mathematics Module 5

Grade 3 Mathematics Module 5: Fractions as Numbers on the Number Line

In this 35-day Grade 3 module, students extend and deepen second grade practice with "equal shares" to understanding fractions as equal partitions of a whole. Their knowledge becomes more formal as they work with area models and the number line.

Eureka Math G3 Fraction Module

Random G3 District Fraction Pacing Guide

Domain 3: Number and Operations - Fractions
Chapter 10: Fractions (12-13 days)

NJ 2016 Student Learning Standards: Mathematics Grade 3

3.NF.1 – Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a/b as the quantity formed by a parts of size 1/b.

3.NF.2 – Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.2a – Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

3.NF.2b – Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.3 – Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

3.NF.3a – Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

3.NF.3b – Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2}$ = 2/4, 4/6, = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

3.NF.3c – Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

3.NF.3d – Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols <, =, >, and justify the conclusions, e.g., by using a visual fraction model.

3.G.2 – Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

How do the shifts play out with respect to fraction instruction?

- Focus – spend more time
- Coherence – pay attention to the meaning of the cluster and the importance of the unit fraction
- Rigor – follow the progression here – the Grade 3 work in fractions should be almost entirely CONCEPTUAL

3.NF Unpacked

Number and Operations—Fractions[5] **3.NF**

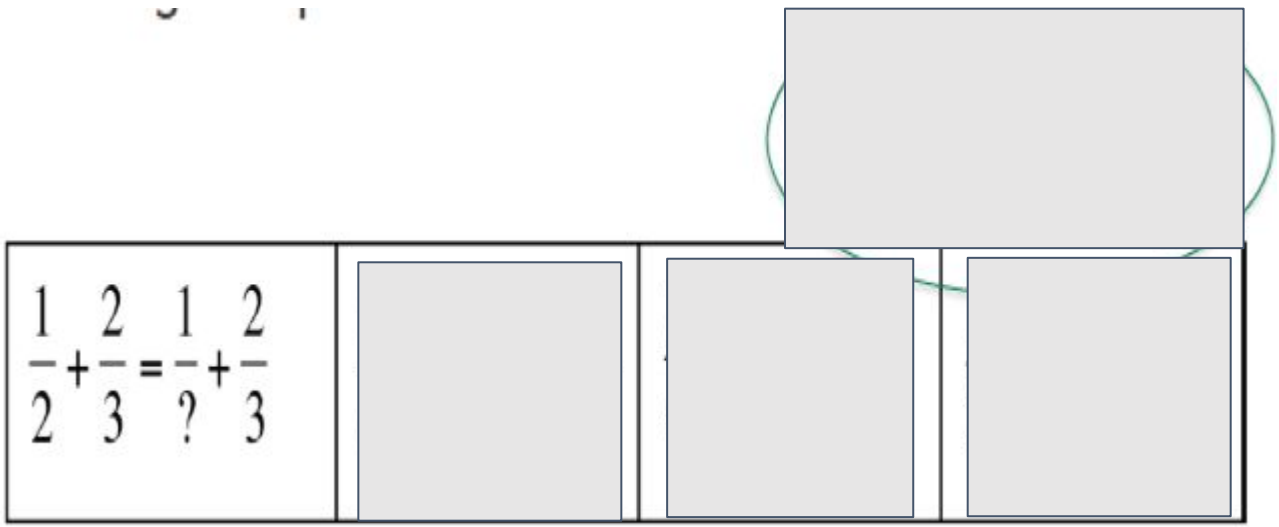
Develop understanding of fractions as numbers.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
   b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Verbs:
understand (x4); represent (x3); recognize (x5); explain (x2); compare (x2); express; record; justify; generate

Number of standards calling for real world (application) problems: zero

What does it mean to extend the work of whole numbers to fractions?



1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.
1.OA.B.3 Apply properties of operations as strategies to add and subtract.

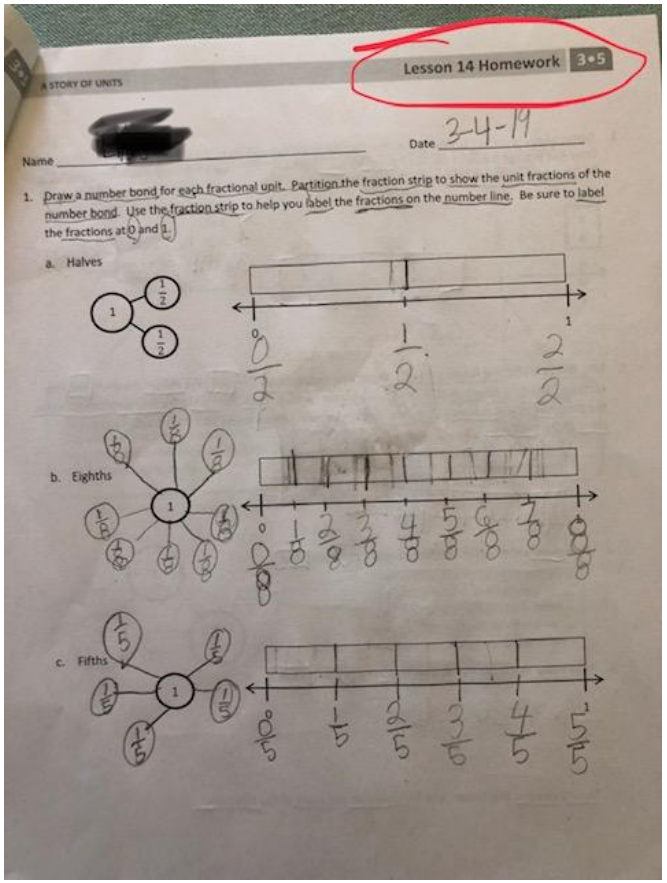
Zoom in on Materials: Lesson 14 of 35

What mathematics is being developed?

What mathematical transition do you see happening?

What mathematics might students have been working on before to get here?

What might be still ahead in the grade 3 work?



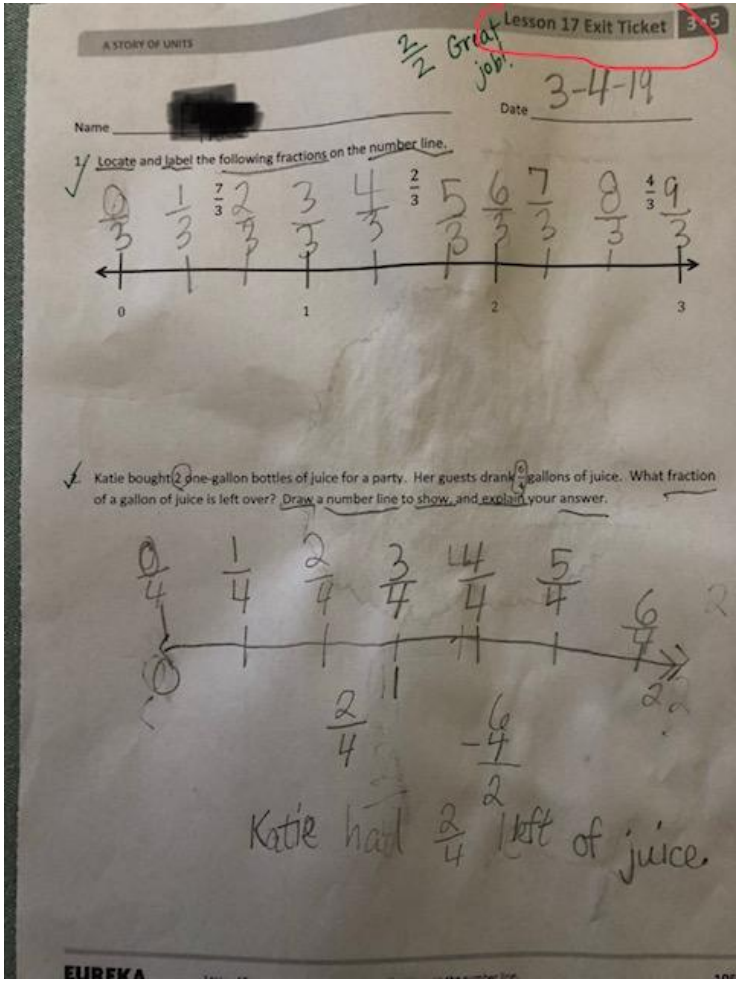
Zoom in on Materials: Lesson 17 of 35

What new mathematical ideas are students working with here?

Are they really “new”?

What is interesting about the student’s approach to problem #2 here? What questions do we have for the student about his/her work?

For problem 2, how might students’ work look different if the last sentence/direction was not there?



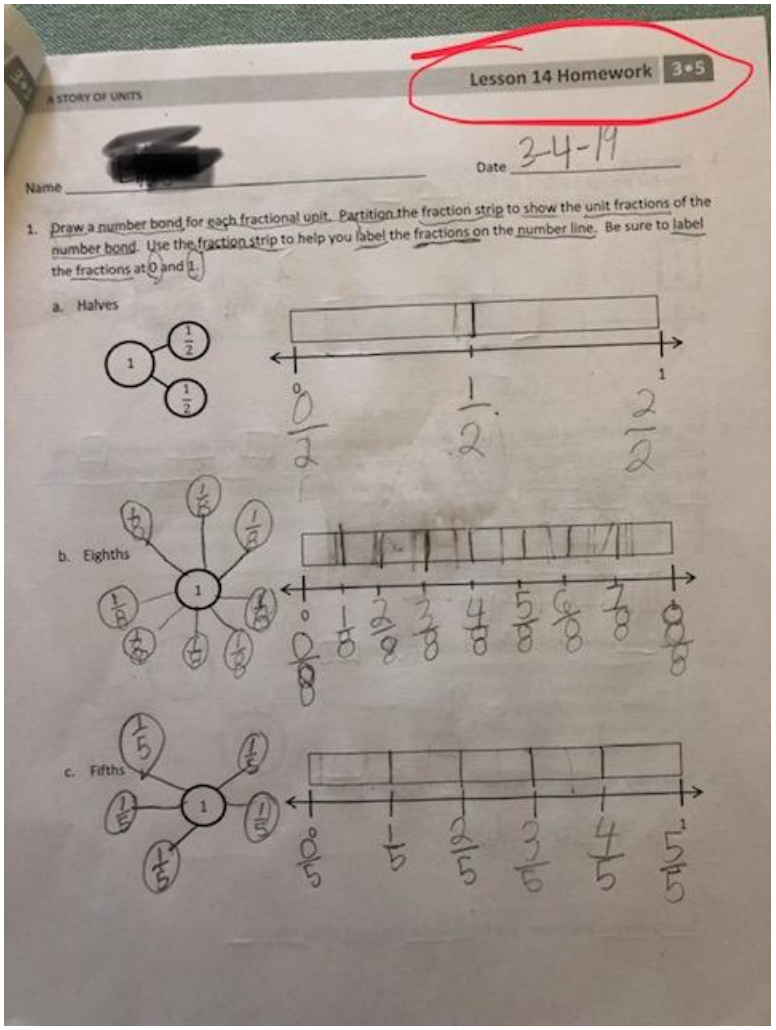
Revisiting the Shift of Coherence

4.NF.B.3 Understand a fraction a/b with a>1 as a sum of fractions 1/b.

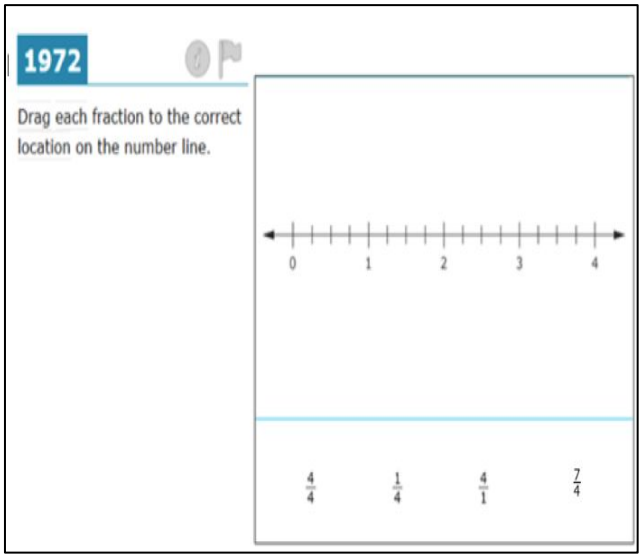
4.NF.B.4a Understand a fraction a/b as a multiple of 1/b.

4.NF.B.4b Understand a multiple of a/b as a multiple of 1/b and use this understanding to multiply a fraction by a whole number.

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.



Assessment of Fractions

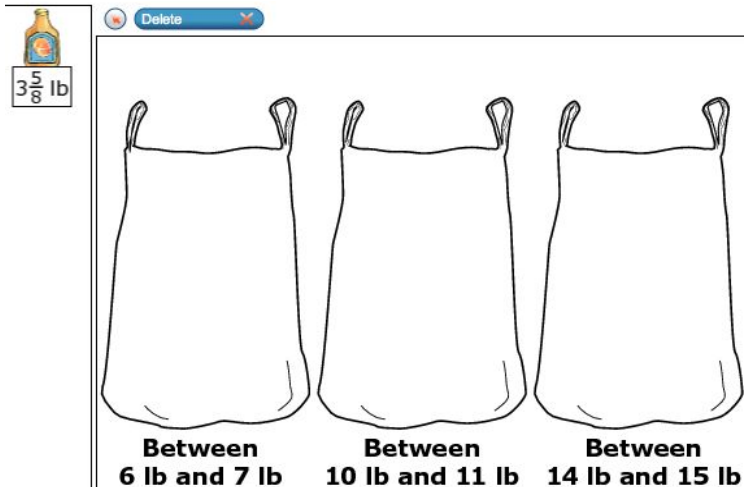


Select **all** the numbers that make this inequality true.

$2\frac{1}{8} > \square + 1 + \frac{1}{8}$

- ☐ $\frac{1}{8}$
- ☐ $\frac{4}{8}$
- ☐ $\frac{10}{8}$
- ☐ $\frac{16}{8}$

Drag juice bottles into each bag to create a bag within the range of weights shown. If you cannot create a bag within the range of weights shown, leave the bag empty.



Drag and drop the expressions into the correct order.

Topic 3:

Multiplication

Topic 3: Multiplication Expressions

- Why is multiplication on the list?
- How does the language of the CCSS influence the way we teach multiplication?
- Grade 3 and Grade 4
  - What has been done?
  - What can we do differently (and better)?

Topic 3: Multiplication

What is 3 x 4?

How might students answer this?

3rd Grade CCSS

CCSS.MATH.CONTENT.3.OA.A.1

Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7.*

CCSS.MATH.CONTENT.3.OA.C.7

Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

What has been done?

Chapter 3 Understand Multiplication

What can we do differently?

Cool-down: Equal Groups Drawings

Jada had 3 bags. Each bag has 5 bracelets in it.

Draw a picture to represent the situation.

Cool-down: Create a Tape and Expression

There were 6 envelopes. Each envelope had 2 notes in it.

1. Draw a tape diagram to represent the situation.
2. Write a multiplication expression to represent the situation

Cool-down: Ducks in a Pond

There are 4 ponds. Each pond has 5 ducks. How many ducks are there altogether? Show or explain your reasoning.

Cool-down: Create a Drawing

The store has 4 boxes. Each box has 10 shirts in it.

1. Draw a tape diagram to represent the situation.

Grade 4 Multiplication of a Fraction

What is $3 \times \frac{3}{4}$?

How might students answer this?

4th Grade CCSS

CCSS.MATH.CONTENT.4.NF.B.4.B

Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*

CCSS.MATH.CONTENT.4.NF.B.4.C

Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

What has been done?

Chapter 7 Add and Subtract Fractions

7.1 Investigate: Join and Separate Fractions
7.2 Write Fractions as Sums
7.3 Add Fractions Using Models
7.4 Subtract Fractions Using Models
7.5 Add and Subtract Fractions
Mid-Chapter Checkpoint
7.6 Rename Fractions and Mixed Numbers
7.7 Add and Subtract Mixed Numbers
7.8 Record Subtraction with Renaming
7.9 Algebra: Fractions and Properties of Addition
7.10 Problem Solving: Multistep Problems with Fractions
Chapter 7 Review/Test
Performance Task

Chapter 8 Multiply Fractions by Whole Numbers

8.1 Investigate: Multiples of Unit Fractions
8.2 Investigate: Multiples of Fractions
Mid-Chapter Checkpoint
8.3 Model Multiplication of a Fraction by a Whole Number
8.4 Multiply a Fraction by a Whole Number
8.5 Problem Solving: Comparison Problems with Multiplication
Chapter 8 Review/Test
Performance Task

Lessons 1–2:	Decompose fractions as a sum of unit fractions using tape diagrams.
Lesson 3:	Decompose non-unit fractions and represent them as a whole number times a unit fraction using tape diagrams.
Lesson 4:	Decompose fractions into sums of smaller unit fractions using tape diagrams.
Lesson 5:	Decompose unit fractions using area models to show equivalence.
Lesson 6:	Decompose fractions using area models to show equivalence.

What can we do differently?

Draft of IM Elementary Curriculum

Grade 4 Unit 2 Section A

4.2.A.1 Represent Equal Groups

4.2.A.2 Represent Equal Groups of Unit Fractions

4.2.A.3 Represent Equal Groups of Fractions with Expressions

4.2.A.4 Find Patterns in Fraction Multiplication

4.2.A.5 Solve Problems involving Equal Groups of Fractions

Grade 4: Instructional Routine

True or False?

$$4 \text{ x } \frac{79}{90} = 5 \text{ x } \frac{79}{90}$$

$$3 \text{ x } \frac{89}{90} = 3 \text{ x } \frac{89}{91}$$

$$3 \text{ x } (2 \text{ x } \frac{54}{90}) = 6 \text{ x } \frac{54}{90}$$

Coaching For Content

- Choose and area of focus (*What is your topic? What do the Standards and Progressions say across grades?)*
- Reflect on current practice (*What do we currently do? How could we do it better?)*
- Choose a task around this topic (*What tasks on the Coherence Map support this topic?*)
- Do Math Together (*What are students building on? addressing? building toward?*)
- Try the task in a classroom (*What did students say and do?*)
- Meet to discuss (*What did we learn about student thinking in relations to this topic? How will this impact my instructin?*)

THANK YOU

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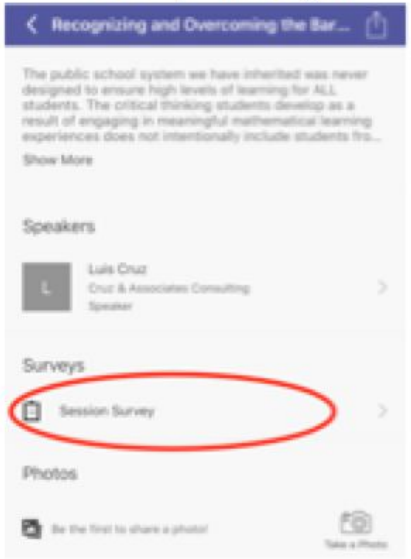

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